The Effect of Noise on Transverse Emittance Growth in the Tevatron[*]

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ABSTRACT: Emittance growth due to noise from a transverse beam feedback system are discussed. A theory for calculating emittance growth rate as a function of the feedback system's measured open loop transfer function is derived. A simple feedback system was installed, measured, and tested in the Fermilab Tevatron, and the emittance growth rate results agree very closely with the theory.

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INTRODUCTION

As particle densities in accelerators and storage rings increase, instabilities arise that dilute emittance or, at worst, actually drive the beam out of the machine. Beam feedback systems can extend the intensity threshold of these instabilities. These systems have pushed the intensity limits of many accelerators beyond their design intensity and have become a necessity for present day operation.

For most beam feedback applications, the dominant design specification is the system gain required to counteract the instability growth rate. This gain is predominantly limited by the delay and dynamic range of the system. A secondary but significant design specification is the amount of noise power that is deposited in the beam. This noise will cause emittance growth over time periods on the order of minutes unless there is some other damping mechanism such as synchrotron radiation. Noise will have a minimal effect on fast cycling machines, but it could have a devastating effect on hadron storage rings that must maintain high luminosity.

Instrumentation for measuring feedback system response functions has become increasingly powerful in the last four years, and feedback designers have taken advantage of this when designing new feedback systems for existing machines. By measuring the system response at low beam intensity, the stability of the system at higher intensities can be extrapolated. The theory and application of this technique are well documented.

This paper documents the extrapolation of noise properties of the feedback system from the system response measurements. First, a theory for determining relative emittance growth rate as a function of the system response is derived. Second, the theory is tested in the Fermilab Tevatron storage ring. The paper will show how the measured relative growth rates compare between feedback loops open and closed, and it will show how closely these rates compare with theory.

<center>THEORY</center>

In this section, we will derive the relative growth rates of the emittance of the beam when the damper loop is open and when it is closed given its open loop response function $G(\omega)H(\omega)$. Referring to Figure 1(a), if the response function in the time domain is $h(t) = u(t)\bar{h}(t)$ where $u(t)$ is the unit step function, then the output y_{out} from noise $n(t)$ is simply the convolution

$$
\left.
\begin{aligned}
y_{\text{out}}(t) &= \int_{-\infty}^{\infty} d\tau \; h(\tau)n(t-\tau) \\
&= \int_{0}^{t} d\tau \; \bar{h}(\tau)n(t-\tau)
\end{aligned}
\right\}
\tag{1}
$$

where we have used $n(t) = 0$ if $t < 0$. Then the mean squared growth $\langle y_{\text{out}}^2 \rangle$ is simply

$$
\left.
\begin{aligned}
\langle y_{\text{out}}^2 \rangle &= \int_{0}^{t} d\tau \int_{0}^{t} d\tau' \; \bar{h}(\tau)\bar{h}^*(\tau')\langle n(t-\tau)n(t-\tau')\rangle \\
&= \int_{0}^{t} d\tau \int_{0}^{t} d\tau' \int_{-\infty}^{\infty} d\omega \; \bar{h}(\tau)\bar{h}^*(\tau')S(\omega)e^{i\omega(\tau-\tau')} \\
&= \int_{-\infty}^{\infty} d\omega \left| \int_{0}^{t} d\tau \; \bar{h}(\tau)e^{i\omega\tau} \right|^2 S(\omega)
\end{aligned}
\right\}
\tag{2}
$$

where $S(\omega)$ is the spectral power density function. $S(\omega)$ is defined to be

$$
P_{\text{av}} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} dt \; x^2(t) = \int_{-\infty}^{\infty} d\omega \; S(\omega)
\tag{3}
$$

and it can be shown (refer to reference 1) that

$$
\left.
\begin{aligned}
\langle n(t-t')n(t-t'')\rangle &= \langle n(0)n(t''-t')\rangle \qquad \text{stationary property} \\
&= \int_{-\infty}^{\infty} d\omega \; S(\omega)e^{-i\omega(t''-t')}
\end{aligned}
\right\}
\tag{4}
$$

For Figure 1(b), when there is a gain element $G(\omega)$ in the system, we can use (3) and Parseval's theorem to show that

$$
\langle y_{\text{open}} \rangle^2 = \int_{-\infty}^{\infty} d\omega \left| \int_{0}^{t} d\tau \; \bar{h}(\tau)e^{i\omega\tau} \right|^2 |G(\omega)|^2 S(\omega)
\tag{5}
$$

Finally for Figure 1(c), when we close the loop,

$$
G(\omega) \to \frac{1}{1 + G(\omega)H(\omega)}
\tag{6}
$$

<center>3</center>

and thus

$$\langle y_{\text{closed}}\rangle^2 = \int_{-\infty}^{\infty} d\omega \left| \int_0^t d\tau \, \bar{h}(\tau)e^{i\omega\tau} \right|^2 \left| \frac{1}{1+G(\omega)H(\omega)} \right|^2 S(\omega) \tag{7}$$

If the observed growth rate when t is large is dominated by the linear part of $\langle y_{\text{open}}\rangle^2$ then

$$d\langle y_{\text{open}}\rangle^2/dt = \text{slope} \left[\int_{-\infty}^{\infty} d\omega \left| \int_0^t d\tau \, \bar{h}(\tau)e^{i\omega\tau} \right|^2 |G(\omega)|^2 S(\omega) \right] \tag{8}$$

where slope[.] is defined to be the slope of the function for large t. Similarly

$$d\langle y_{\text{closed}}\rangle^2/dt = \text{slope} \left[\int_{-\infty}^{\infty} d\omega \left| \int_0^t d\tau \, \bar{h}(\tau)e^{i\omega\tau} \right|^2 \left| \frac{1}{1+G(\omega)H(\omega)} \right|^2 S(\omega) \right] \tag{9}$$

If we assume $S(\omega) = \text{constant}$ and $G(\omega) = g$, where g is independent of ω, then the relative growth is

$$\frac{d\langle y_{\text{closed}}^2\rangle/dt}{d\langle y_{\text{open}}^2\rangle/dt} = \frac{\text{slope}\left[\int_{-\infty}^{\infty} d\omega \left| \frac{\int_0^t d\tau \, \bar{h}(\tau)e^{i\omega\tau}}{1+gH(\omega)} \right|^2 \right]}{\text{slope}\left[\int_{-\infty}^{\infty} d\omega \left| \int_0^t d\tau \, \bar{h}(\tau)e^{i\omega\tau} \right|^2 g^2 \right]} \tag{10}$$

and

$$\frac{d\langle y_{\text{closed}}^2\rangle/dt}{d\langle y_{\text{closed}}^2\rangle/dt|_{g=0}} = \frac{\text{slope}\left[\int_{-\infty}^{\infty} d\omega \left| \frac{\int_0^t d\tau \, \bar{h}(\tau)e^{i\omega\tau}}{1+gH(\omega)} \right|^2 \right]}{\text{slope}\left[\int_{-\infty}^{\infty} d\omega \left| \int_0^t d\tau \, \bar{h}(\tau)e^{i\omega\tau} \right|^2 \right]} \tag{11}$$

Equation (11) is the equation which we will use to fit the measured data shown in the next section.

EXPERIMENT

The simplified block diagram of our narrow band damper system is shown in Figure 2. This block diagram is an example of the system shown in Figure 1(c), Thus, we can use the previously discussed theoretical results to analyze the experimental data.

When G is set to zero and only noise is fed into the beam at 150 GeV, we see that the vertical beam size grows as expected. This is shown in Figure 3. In fact the growth of $\langle y^2_{\text{closed}}\rangle|_{G=0}$ is linear as shown in Figure 4. Now, when the loop is closed and $G \approx 5.5$ at 18.327 kHz, the growth rate is reduced by a factor of 3 from $(0.71 \pm 0.05) \times 10^{-3}$ mm^{2}s^{-1} to $(0.24 \pm 0.05) \times 10^{-3}$ mm^{2}s^{-1}.

We plot the measured relative growth rates and the theoretical relative growth rates in Figure 5. The theoretical growth rate was calculated using (11) with the measured $G(\omega)H(\omega)$ shown in Figure 6. It is important to notice that there are no free parameters in the theory, i.e. the theory is not fitted to the data. The goodness of fit criteria is given by

$$\chi^2_\nu = \frac{1}{\nu} \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left(\frac{d\langle y^2_{\text{expt}}\rangle_i}{dt} - \frac{d\langle y^2_{\text{theory}}\rangle_i}{dt} \right)^2 \tag{12}$$

where $\nu = N - 1$ and N is the number of data points. Using (12), the goodness of fit of our theory to the measured data is 1.3. According to Bevington[4] χ^2_ν should be about 1 and less than 1.5 for a good fit. Therefore, using Bevington's criterion, we conclude that our theory is a good fit to the observations. As a comparison, we also show in this plot a linear least squares fit constrained to pass through $(0, 1)$ of the data. The linear least squares plot has a $\chi^2_\nu \approx 3$.

Finally the measured absolute growth rate when the loop is open versus the spectral power density is shown in Figure 7. We see that the the growth rate is linear w.r.t. spectral power density. The least squares fit gives the emittance growth rate per spectral power

density to be $(0.55 \times 10^{-2})\pi$ (mm·mrad·s^{-1})/(W/kHz) or 19.8π (mm·mrad·hr^{-1})/(W/kHz) over the narrow band of 2 kHz which encloses all the tune lines centred about 18.847 kHz. Using this number and assuming that the maximum emittance growth rate is 0.2π mm·mrad ·hr^{-1} allowed in the Tevatron, then the maximum noise density that we can have in the narrow band damper system is 10 mW/kHz. For a broadband system of 10 MHz, we have approximately 425 tune bands in this bandwidth, then the maximum noise density that we can allow in the broadband system is 24 μW/kHz if we assume that each band contributes linearly to the growth rate.

CONCLUSION

We have demonstrated how we can calculate the growth rate when we are given the open loop response. This observed relative growth rate agrees with the theoretical relative growth rate. Therefore, if we are given the open loop response of the damper system, we can calculate what the expected growth rate of the beam. Finally, from our measurements of the absolute growth rate we can put an upper limit on the noise density that we are allowed for the damper design.

REFERENCES

[1] J.S. Bendat, *Principles and Applications of Random Noise Theory*, John Wiley & Sons, 1958.

[2] A.W. Chao, *Physics of Collective Beam Instabilites in High Energy Accelerators*, Wiley Interscience, 1993.

[3] D. Möhl, *Stochastic Cooling*, Cern Accelerator School, Advanced Accelerator Physics, CERN 87-03, pp. 453-533, 1987.

[4] P.R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill Book Company,1969.

Figure 1 Here are the three systems that will be considered in this section: (a) The beam only system where the growth rate is linear w.r.t. time. (b) The beam with noise after going through G which we call *gain*. However, this gain is really the frequency response of the electronics and is represented here as a general linear, time-invariant system. (c) The system when we close the loop. In this case, we add the noise after G, but in practice, we can add in the noise at any other point and do a similar analysis.

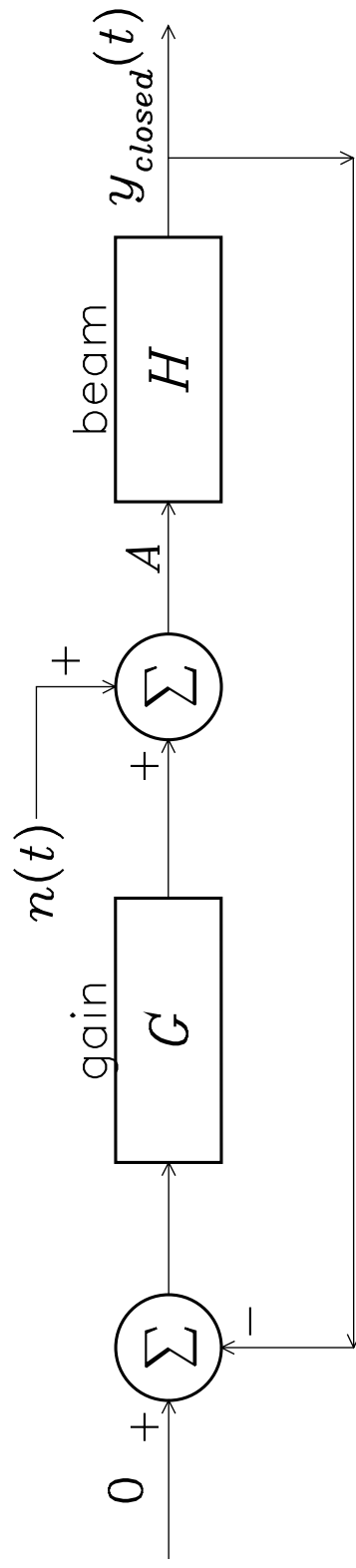(a) Beam only
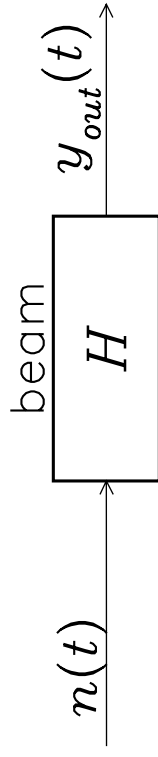
(b) Open Loop

(c) Closed Loop

9
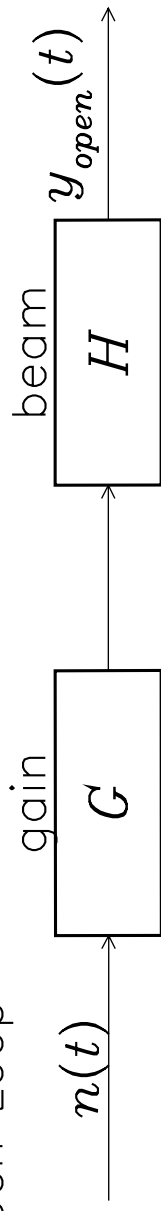
Figure 2   This figure shows a simplified block diagram of our narrow band damper. We define G to be pure gain and everything else is in H. We can compare this block diagram to Figure 1(c) to see that they are the same system.
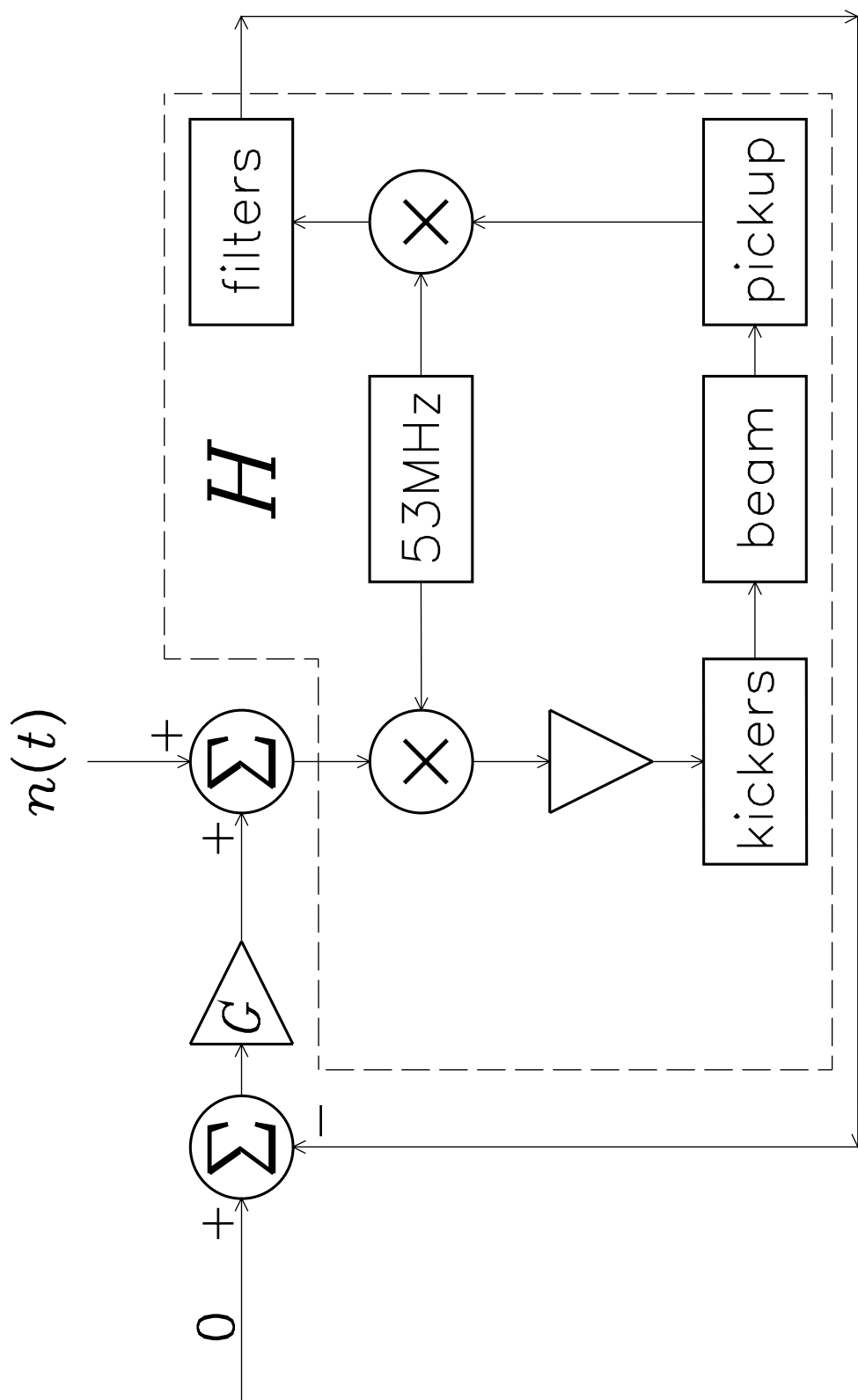
Figure 3 This graph shows how the vertical beam size typically grows when a noisy vertical kicker is used to excite the beam. In this case, the noise density $S_0 = 1.25$ W/kHz between 17.417 kHz and 19.417 kHz. The three lines that are shown are (a) the beam current (IBEAM) in units of 10^{12} particles, (b) vertical beam size sigma (FWVSIG) in units of mm and (c) horizontal beam size sigma (FWHSIG) in units of mm. The horizontal scale is in seconds. The steps which are shown in FWHSIG and FWVSIG is from taking data once every 15 seconds.
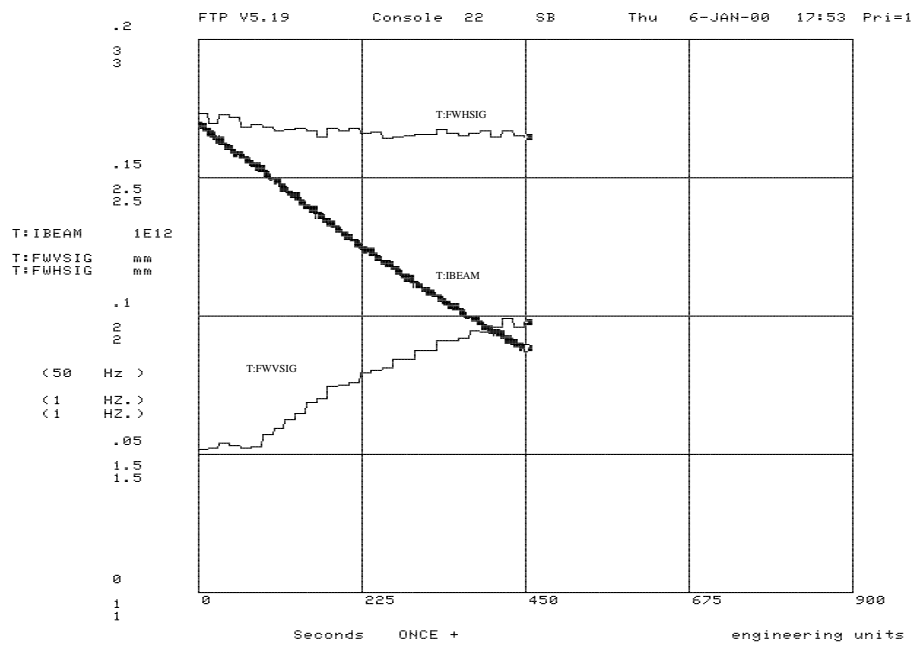
```
.2
3
3

                                                              T:FWHSIG

.15
2.5
2.5

T:IBEAM    1E12

T:FWVSIG    mm
T:FWHSIG    mm
.1                                                     T:IBEAM
2
2

(50    Hz )                             T:FWVSIG
(1     HZ.)
(1     HZ.)
.05
1.5
1.5


0
1
1
        0          225          450          675          900

        Seconds    ONCE +                      engineering units
```

13

Figure 4 This figure shows the result of processing the raw data shown in Figure 3 after going through our fitting routine which corrects for the beam current decay. It is clear from here that the growth rate is linear which satisfies the condition required in the theory. At 1.25 W/kHz, the growth rate is $(0.71 \pm 0.05) \times 10^{-3}$ mm^{2}s^{-1}. When the loop is closed, the growth rate is reduced by a factor of 3 to $(0.24 \pm 0.05) \times 10^{-3}$ mm^{2}s^{-1}. Again, there are steps shown in the processed data because data is taken only once every 15 seconds.

Growth in Beam Size

$\langle y^2 \rangle$ mm^2
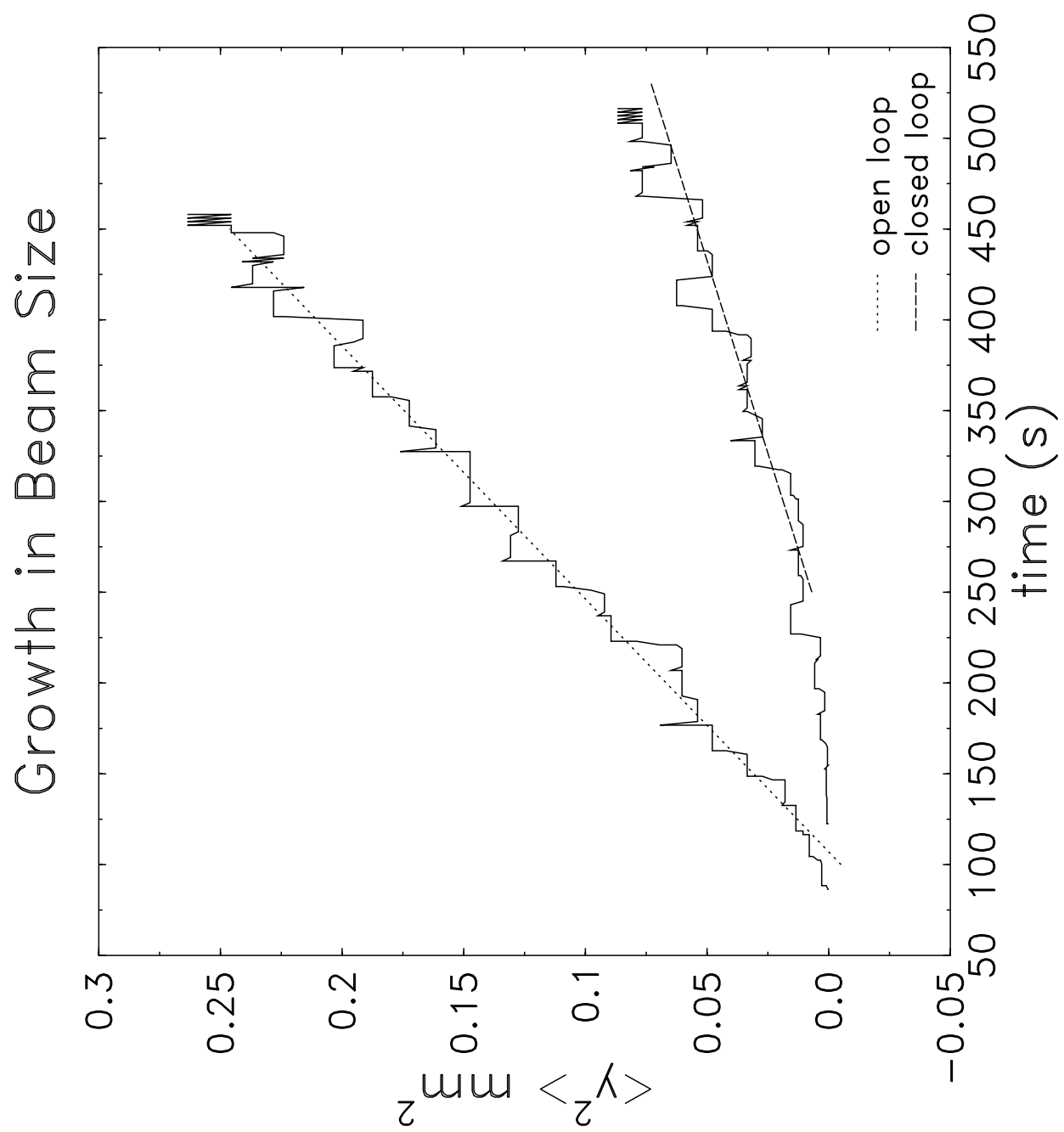
time (s)

open loop
closed loop

15

Figure 5   The measured relative growth rate versus the relative gain is shown in the top graph. The theoretical curve is calculated using the the open loop response shown in Figure 6. A linear least squares fit of the data which must pass through the point $(0, 1)$ is superimposed for comparison.
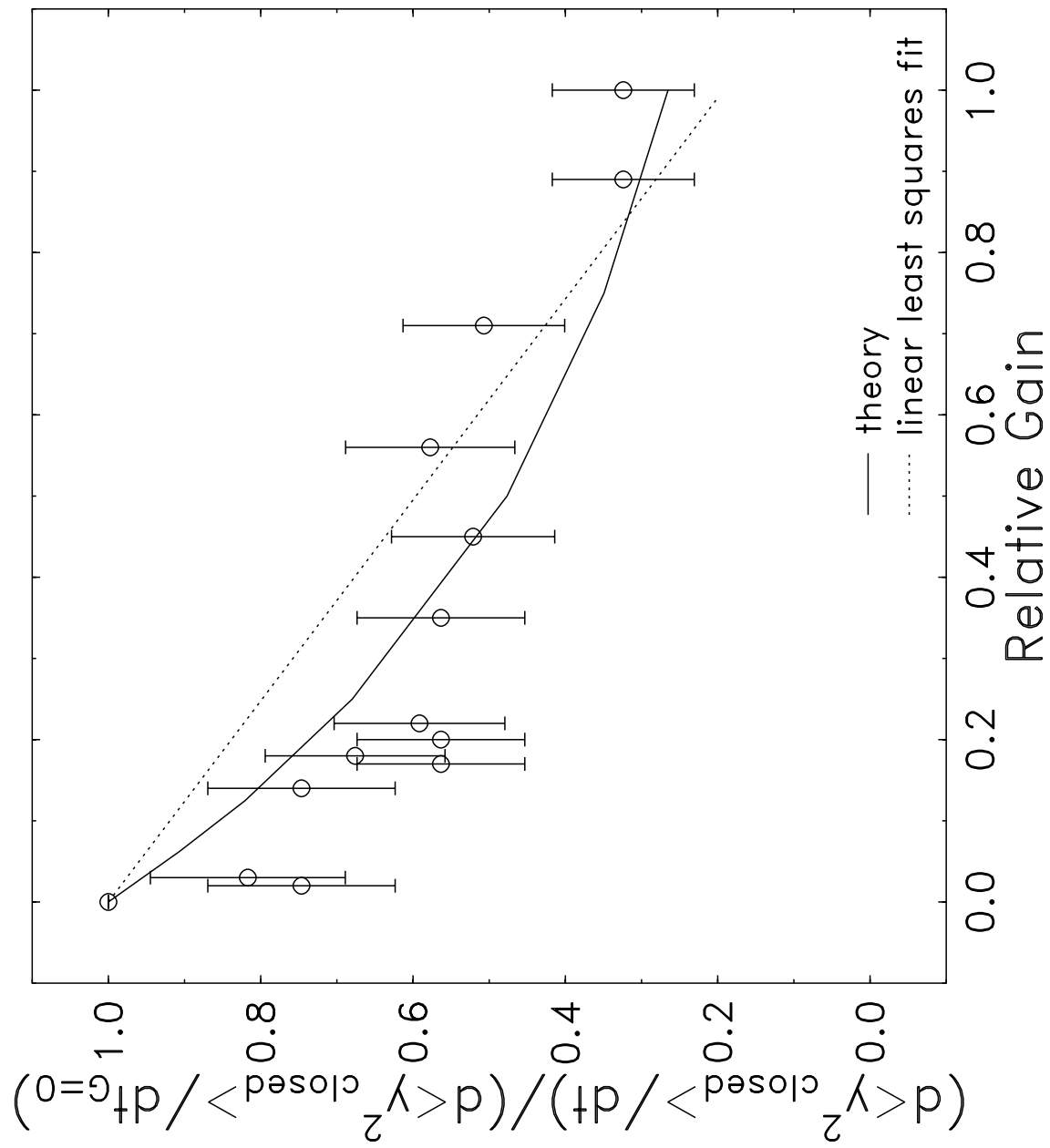
Relative Growth Rate vs Relative Gain

Figure 6   This is the measured open loop response of the system i.e. $G(\omega)H(\omega)$.
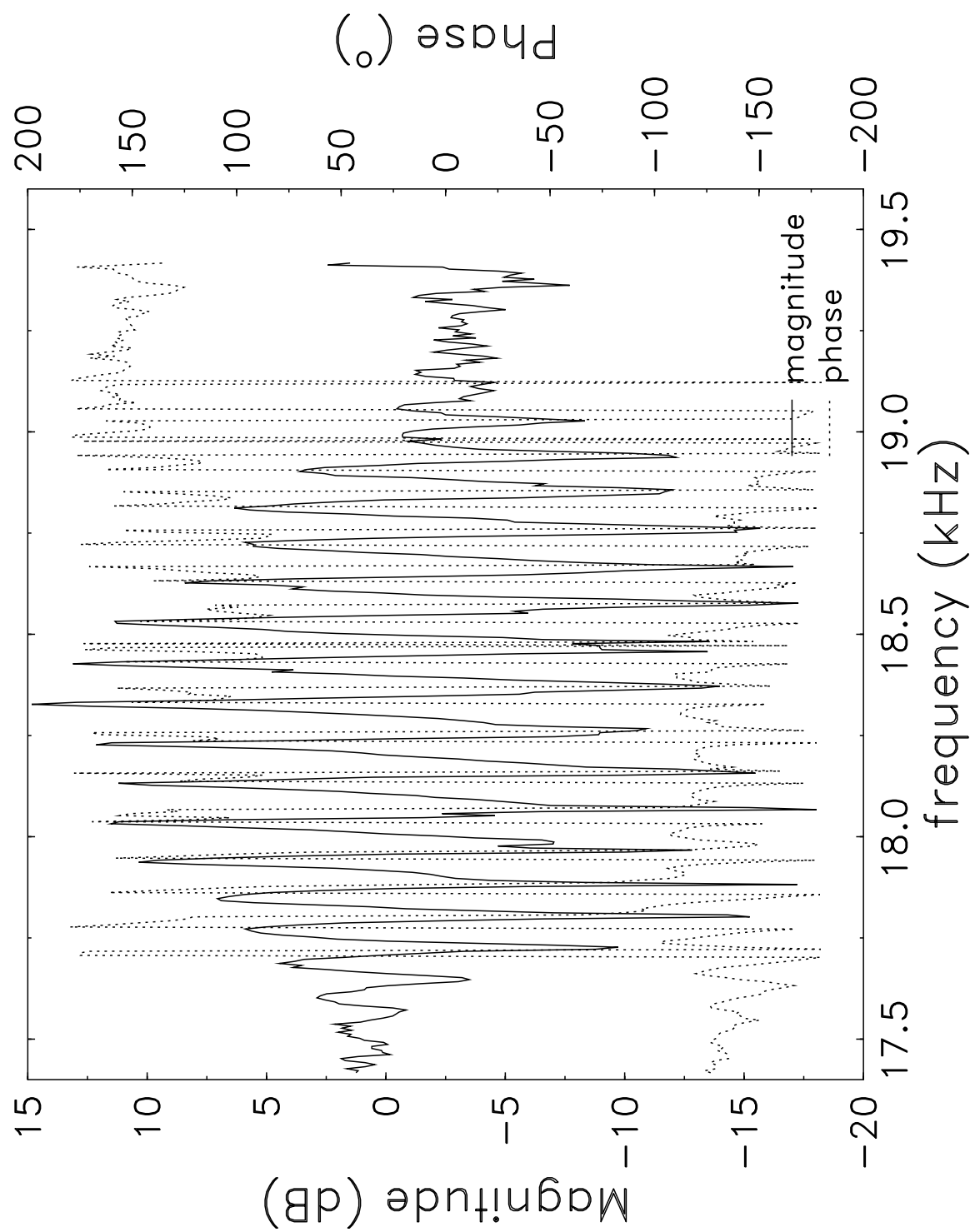
Open Loop Response

Figure 7 The absolute growth rate at 150 GeV when the loop is open versus the spectral power density is linear. The noise bandwidth is fixed at 2 kHz centred about 18.847 kHz. The least squares fit is $d\langle y^2\rangle/dt = (0.52 \times 10^{-3})S$ $(\mathrm{mm^2 s^{-1}})$. The vertical emittance is calculated using $d\epsilon_y/dt = \pi 6\gamma(d\langle y^2\rangle/dt)/\beta_y = \pi 10.57 d\langle y^2\rangle/dt$ $\mathrm{mm{\cdot}mrad{\cdot}s^{-1}}$ with $\gamma = 159.9$ and $\beta_y = 90.8$ m.

Growth from Noise